Abstracts of the March 23, 2018 session of the Paris-London Analysis Seminar

Laurent Baratchart (Inria, Sophia-Antipolis,
Topics in Rational and Meromorphic Approximation.

Abstract. We consider rational and meromorphic approximation with n poles to analytic functions on a compact subset of the domain of analyticity. We shall recast this issue as optimal discretization of logarithmic potentials, and stress some applications to identification and elliptic inverse problems. We will discuss the asymptotic behaviour of poles, when n gets large, for best approximants to certain classes of functions, in connection with interpolation theory, the Adamjan-Arov-Krein theory, and extremal geometric problems from potential theory. We will also speak on error rates along with constructive aspects, and raise some questions in higher dimension.

Leonid Bogachev (University of Leeds),
Liouville-type theorems for the archetypal equation with rescaling.

Abstract. In this talk, we consider a linear functional-integral equation

$$y(x) = \iint_{\mathbb{R}^2} y(a(x-b))\,\mu(\mathrm{d}a, \mathrm{d}b), \qquad x \in \mathbb{R},$$

where μ is a probability measure on $\mathbb{R}^2$; equivalently, $y(x) = \mathbb{E}\{y(\alpha(x-\beta))\}$, with random (α, β) and $\mathbb{E}$ denoting expectation. This is a rich source of various functional and functional-differential equations with rescaling (hence the name *archetypal*), exemplified by the integrated Cauchy equation $y(x) = \mathbb{E}\{y(x-\beta)\}$ and the functional-differential ('pantograph') equation $y'(x) + y(x) = \mathbb{E}\{y(\alpha(x-\gamma))\}$.
Interpreting solutions $y(x)$ as harmonic functions of the associated Markov chain (X_n), we discuss Liouville-type theorems asserting that any bounded continuous solution is constant. For instance, in the case $\alpha \equiv 1$ this is the celebrated Choquet–Deny theorem. In general, results crucially depend on the criticality parameter $K := \mathbb{E}\{\ln|\alpha|\}$; e.g., if $K < 0$ then a Liouville theorem is always true, but the case $K \geq 0$ is more interesting (and difficult). The proofs utilize the iterated equation $y(x) = \mathbb{E}\{y(X_\tau)|X_0 = x\}$ (with a suitable stopping time τ) due to Doob's optional stopping theorem applied to the martingale $y(X_n)$.
This is joint work with Gregory Derfel (Beer Sheva) and Stanislav Molchanov (UNC-Charlotte).

Victor Chulaevsky (Université de Reims),
Smoothness of density of states and localization under long-range interactions with arbitrary disorder: New challenges.

Abstract. Numerous works on spectral and dynamical properties of disordered media, in mathematical and theoretical physics, are based on the assumption of local regularity of the disorder. It was shown in the seminal paper by F. Wegner (1981) that if a finite matrix has independent random diagonal elements with a Lipschitz continuous distribution and its off-diagonal part is non-random, then its eigenvalue distribution is also Lipschitz continuous. The Wegner estimate has been generalized in various directions and applied to many discrete and continuous random Hamiltonians; it is a crucial component of the rigorous proofs of Anderson localization. However, the regularity of the original, local disorder is questionable from the physical perspective. An extension to the models with very singular disorder (e.g., Bernoulli-distributed amplitudes in the so-called continuous alloy models) is much more recent, and the available techniques, surprisingly, fail to apply to the discrete (e.g., lattice) systems.
In contrast to a majority of mathematical works, we consider in the talk alloy models with physically more realistic (infinite-range) site potentials and show that classical results on infinite convolutions of singular probability measures, a subject that has been actively studied in harmonic analysis, probability theory/statistics, and dynamical systems since early 20th century, shed a light on this hard problem and allow one to prove in many cases an infinite smoothness of the finite-volume eigenvalue distribution measure. At the same time, the first results in this direction also raise new interesting and challenging questions.

Stephen Power (University of Lancaster),
Crystal flexibility: methods from Analysis and Commutative Algebra.

Abstract. The stability of crystal lattices was considered by Max Born and coworkers in the 1940s by essentially linear (small vibration) methods for which "it is not necessary to consider the complete elastic spectrum with its innumerable proper frequencies". The analysis of such mechanical modes (a.k.a. zero modes, rigid unit modes) is an ongoing research topic in both mathematics (infinitesimal and combinatorial rigidity) and condensed-matter science (surface modes and topological modes). I shall survey some mathematical rigidity theory and I hope to indicate some open problems and new methods from analysis and commutative algebra. In particular using algebraic spectral synthesis methods necessary and sufficient conditions have been obtained for the first-order rigidity of a crystallographic bar-joint framework (Kastis and Power, 2018).