ADVANCES IN THE THEORY OF MULTI-DIMENSIONAL SHOCK WAVES

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Abstract

Roughly, a shock in a quasilinear hyperbolic PDE solution is a mild singularity such that one of its derivatives blows up, though the solution itself remains bounded. Importantly, the mild nature of the singularity opens the door to the possibility that under suitable selection criteria, the solution might be continued uniquely as a weak solution, past the shock. While the rigorous $1D$ theory is in a mature state due to the availability of well-posedness results for BV initial data, multi-dimensional hyperbolic PDEs are typically ill-posed in BV. Consequently, the theory of multi-dimensional shocks is with permeated with fundamental open problems, many with deep ties to geometry. Despite the difficulties in higher dimensions, for specific systems, including the compressible Euler equations and relativistic Euler equations in $3D$, there has been dramatic progress in the last 15 years, starting with Christodoulou's 2007 monograph on shock formation in irrotational solutions. In this talk, after providing a brief introduction to the $1D$ problem, I will give a non-technical description of recent advances in multi-dimensions, with a focus on the multi-dimensional compressible Euler equations with vorticity and entropy. Many recent results are based on a new formulation of compressible Euler flow exhibiting miraculous geo-analytic structures and regularity properties, and the analysis fundamentally relies on nonlinear geometric optics. In particular, I will describe my recent series of works on the $3D$ compressible Euler equations with vorticity and entropy which, for open sets of initial data, reveal the full structure of the maximal classical development, including the full structure of the singular set as well the emergence of a Cauchy horizon from the singularity. Finally, time permitting, I will discuss the many open problems in the field. Various aspects of this program are joint with L. Abbrescia, J. Luk, and M. Disconzi.