SPECTRAL ANALYSIS OF JACOBI OPERATORS AND ASYMPTOTIC BEHAVIOR OF ORTHOGONAL POLYNOMIALS

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We find and discuss asymptotic formulas for orthonormal polynomials $P_n(z)$ with recurrence coefficients a_n, b_n. Our main goal is to consider the case where off-diagonal elements $a_n \to \infty$ as $n \to \infty$. Formulas obtained are essentially different for relatively small and large diagonal elements b_n.

Our analysis is intimately linked with spectral theory of Jacobi operators J (three-diagonal semi-infinite matrices) with coefficients a_n, b_n and a study of the corresponding second order difference equations. We introduce the Jost solutions $f_n(z)$ of such equations by a condition for $n \to \infty$ and suggest an Ansatz for them playing the role of the semiclassical Liouville-Green Ansatz for solutions of the Schrödinger equation. This allows us to study the spectral structure of Jacobi operators and their eigenfunctions $P_n(z)$ by traditional methods of spectral theory developed for differential equations. The results obtained depend crucially on a rate of growth of the Jacobi coefficients a_n and b_n. In particular, we find asymptotic formulas generalizing the classical formulas for the Hermite and Laguerre polynomials.